Exercise 22

Compute Δy and dy for the given values of x and $dx = \Delta x$. Then sketch a diagram like Figure 5 showing the line segments with lengths dx, dy, and Δy.

$$y = e^x, \quad x = 0, \quad \Delta x = 0.5$$

Solution

Compute the derivative of y.

$$\frac{dy}{dx} = \frac{d}{dx}(e^x)$$

$$= e^x$$

Consequently, the differential of $y = e^x$ is

$$dy = e^x \, dx,$$

so when $x = 0$ and $\Delta x = dx = 0.5$,

$$dy = e^0(0.5) = 0.5$$

$$\Delta y = y(0 + 0.5) - y(0) = e^{0.5} - e^0 \approx 0.648721.$$

The function is plotted below along with its tangent line at $x = 0$.